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Supersymmetric black hole solutions with R^2-interactions *

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Abstract: We present a class of static supersymmetric multi-center black hole solutions arising in four-dimensional $N = 2$ supergravity theories with terms quadratic in the Weyl tensor. We also comment on possible corrections to the metric on the moduli space of these black holes solutions.

1. Introduction

In this note we briefly describe a class of extremal static multi-center black hole solutions arising in four-dimensional $N = 2$ supergravity theories with terms quadratic in the Weyl tensor. These configurations preserve $N = 1$ supersymmetry. They are determined in terms of harmonic functions associated with the electric and magnetic charges carried by the black holes. We refer to an upcoming publication [1] for a detailed description of the construction of these solutions.

2. Supersymmetry transformation rules

The $N = 2$ supergravity theories that we consider are based on vector multiplets and hypermultiplets coupled to the supergravity fields and contain the standard Einstein-Hilbert action as well as terms quadratic in the Riemann tensor. To describe such theories in a transparent way we make use of the superconformal multiplet calculus [2], which incorporates the gauge symmetries of the $N = 2$ superconformal algebra. The corresponding high degree of symmetry allows for the use of relatively small field representations. One is the Weyl multiplet, whose fields comprise the gauge fields corresponding to the superconformal symmetries and a few auxiliary fields. The other are abelian vector multiplets and hypermultiplets, as well as a general chiral supermultiplet. The latter will be treated as independent in initial stages of the analysis but at the end will be expressed in terms of the fields of the Weyl multiplet. Some of the additional (matter) multiplets will provide compensating fields which are necessary in order that the superconformal action becomes gauge equivalent to a Poincaré supergravity theory. The compensating fields bridge the deficit in degrees of freedom between the Weyl multiplet and the Poincaré supergravity multiplet. For instance, the graviphoton, represented by an abelian vector field in the Poincaré supergravity multiplet, is provided by an $N = 2$ superconformal vector multiplet.

It is possible to analyze the conditions for residual $N = 1$ supersymmetry directly in this superconformal setting, postponing a transition to Poincaré supergravity till the end. This implies in particular that our intermediate results are subject to local scale transformations. Only towards the end we will convert to expressions that are scale invariant. We will use this strategy in the following in order to construct black hole solutions with residual $N = 1$ supersymmetry. This is exactly the same strategy we employed when we determined $N = 2$ supersymme-

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tric backgrounds in the presence of R^2-interactions [3].

The superconformal algebra contains general-coordinate, local Lorentz, dilatation, special conformal, chiral SU(2) and U(1), supersymmetry (Q) and special supersymmetry (S) transformations. The gauge fields associated with general-coordinate transformations (e_μ^a), dilatations (b_μ), chiral symmetry ($\mathcal{V}_\mu{}^i{}_j, A_\mu$) and Q-supersymmetry (ψ_μ^i), are realized by independent fields. The remaining gauge fields of Lorentz (ω_μ^{ab}), special conformal (f_μ^a) and S-supersymmetry transformations (ϕ_μ^i) are dependent fields. They are composite objects, which depend in a complicated way on the independent fields [2]. The corresponding curvatures and covariant fields are contained in a tensor chiral multiplet, which comprises $24+24$ off-shell degrees of freedom; in addition to the independent superconformal gauge fields it contains three auxiliary fields: a Majorana spinor doublet χ^i, a scalar D and a selfdual Lorentz tensor T_{abij} (where $i, j, \ldots$ are chiral SU(2) spinor indices). We summarize the transformation rules for some of the independent fields of the Weyl multiplet under Q- and S-supersymmetry and under special conformal transformations, with parameters ϵ^i, η^i and $\Lambda_{\rm K}^a$, respectively,

$$
\begin{aligned}
\delta e_\mu{}^a &= \bar\epsilon^i\gamma^a\psi_{\mu i} + \text{h.c.}\,,\\
\delta\psi_\mu^i &= 2\mathcal{D}_\mu\epsilon^i - \tfrac{1}{8}T^{ab\,ij}\gamma_{ab}\,\gamma_\mu\epsilon_j - \gamma_\mu\eta^i\,,\\
\delta b_\mu &= \tfrac12\bar\epsilon^i\phi_{\mu i} - \tfrac34\bar\epsilon^i\gamma_\mu\chi_i - \tfrac12\bar\eta^i\psi_{\mu i} + \text{h.c.}\\
&\quad +\Lambda_{\rm K}^a\, e_\mu^a\,, \qquad\qquad (2.1)\\
\delta A_\mu &= \tfrac12 i\bar\epsilon^i\phi_{\mu i} + \tfrac34 i\bar\epsilon^i\gamma_\mu\chi_i + \tfrac12 i\bar\eta^i\psi_{\mu i} + \text{h.c.}\,,\\
\delta T_{ab}^{ij} &= 8\bar\epsilon^{[i}R(Q)_{ab}^{j]}\,,\\
\delta\chi^i &= -\tfrac{1}{12}\gamma_{ab}\not{D}T^{ab\,ij}\epsilon_j + \tfrac16 R(\mathcal{V})_{ab}{}^i{}_j\,\gamma^{ab}\epsilon^j\\
&\quad -\tfrac13 iR(A)_{ab}\gamma^{ab}\epsilon^i + D\,\epsilon^i + \tfrac{1}{12}T_{ab}^{ij}\gamma^{ab}\eta_j\,,
\end{aligned}
$$

where $\mathcal{D}_\mu$ are derivatives covariant with respect to Lorentz, dilatational, U(1) and SU(2) transformations, whereas D_μ are derivatives covariant with respect to *all* superconformal transformations. The quantities $R(Q)_{\mu\nu}^i$, $R(A)_{\mu\nu}$ and $R(\mathcal{V})_{\mu\nu}{}^i{}_j$ are supercovariant curvatures related to Q-supersymmetry, U(1) and SU(2) transformations. We suppress terms of higher order in the fermions throughout this paper, as we will be dealing with a bosonic background.

Let us now turn to the abelian vector multiplets, labelled by an index $I=0,1,\ldots,n$. For each value of the index I, there are $8+8$ off-shell degrees of freedom, residing in a complex scalar X^I, a doublet of chiral fermions Ω_i^I, a vector gauge field W_μ^I, and a real SU(2) triplet of scalars Y_{ij}^I. Under Q- and S-supersymmetry the fields X^I and Ω_i^I transform as follows:

$$
\begin{aligned}
\delta X^I &= \bar\epsilon^i\Omega_i^I\,,\\
\delta\Omega_i^I &= 2\not{D}X^I\epsilon_i + \tfrac12\varepsilon_{ij}(F_{\mu\nu}^{-I} - \tfrac14\varepsilon_{kl}T_{\mu\nu}^{kl}\bar X^I)\gamma^{\mu\nu}\epsilon^j\\
&\quad +Y_{ij}^I\epsilon^j + 2X^I\eta_i\,, \qquad\qquad (2.2)
\end{aligned}
$$

where the quantity $F_{\mu\nu}^{-I}$ denotes the anti-selfdual part of the abelian field strength $F_{\mu\nu}^I = 2\partial_{[\mu}W_{\nu]}^I$.

The covariant quantities of the vector multiplet constitute a reduced chiral multiplet. A general chiral multiplet comprises $16+16$ off-shell degrees of freedom and carries an arbitrary Weyl weight w (corresponding to the Weyl weight of its lowest component). The covariant quantities of the Weyl multiplet also constitute a reduced chiral multiplet, denoted by W^{abij}, whose lowest-θ component is the tensor T^{abij}. From this multiplet one may form a scalar (unreduced) chiral multiplet $W^2 = [W^{abij}\,\varepsilon_{ij}]^2$ which has Weyl and chiral weights $w=2$ and $c=-2$, respectively [4].

In the following, we will also allow for the presence of an arbitrary chiral background superfield [5], whose component fields will be indicated with a caret. We denote its bosonic component fields by $\hat A$, $\hat B_{ij}$, $\hat F_{ab}^-$ and by $\hat C$. Here $\hat A$ and $\hat C$ denote complex scalar fields, appearing at the θ^0- and θ^4-level of the chiral background superfield, respectively, while the symmetric complex SU(2) tensor $\hat B_{ij}$ and the anti-selfdual Lorentz tensor $\hat F_{ab}^-$ reside at the θ^2-level. The fermion fields at level θ and θ^3 are denoted by $\hat\Psi_i$ and $\hat\Lambda_i$. Under Q- and S-supersymmetry $\hat A$ and Ψ_i transform as

$$
\begin{aligned}
\delta\hat A &= \bar\epsilon^i\hat\Psi_i\,,\\
\delta\hat\Psi_i &= 2\not{D}\hat A\epsilon_i + \tfrac12\varepsilon_{ij}\hat F_{ab}\gamma^{ab}\epsilon^j + \hat B_{ij}\epsilon^j\\
&\quad +2w\hat A\eta_i\,, \qquad\qquad (2.3)
\end{aligned}
$$

where w denotes the Weyl weight of the background superfield. Eventually this multiplet will be identified with W^2 in order to generate the R^2-terms in the action. This identification implies the following relations [4], which we will

need in due time,

$$\begin{aligned}
\hat{A} &= (\varepsilon_{ij}\, T^{ij}_{ab})^2\,, \\
\hat{\Psi}_i &= 16\,\varepsilon_{ij} R(Q)^j_{ab}\, T^{klab}\,\varepsilon_{kl}\,, \\
\hat{B}_{ij} &= -16\,\varepsilon_{k(i} R(\mathcal{V})^k{}_{j)ab}\, T^{lmab}\,\varepsilon_{lm} \\
&\quad -64\,\varepsilon_{ik}\varepsilon_{jl}\bar{R}(Q)^k_{ab} R(Q)^{l\,ab}\,, \\
\hat{F}^{-ab} &= -16\,\mathcal{R}(M)_{cd}{}^{ab}\, T^{klcd}\,\varepsilon_{kl} \\
&\quad -16\,\varepsilon_{ij}\,\bar{R}(Q)^i_{cd}\gamma^{ab} R(Q)^{j\,cd}\,, \\
\hat{\Lambda}_i &= 32\,\varepsilon_{ij}\,\gamma_{ab} R(Q)^j_{cd}\,\mathcal{R}(M)_{cd}{}^{ab} \\
&\quad +16\,(\mathcal{R}(S)_{ab\,i} + 3\gamma_{[a} D_{b]}\chi_i)\, T^{klab}\,\varepsilon_{kl} \\
&\quad -64\, R(\mathcal{V})_{ab}{}^k{}_i\,\varepsilon_{kl}\, R(Q)^l_{ab}\,, \\
\hat{C} &= 64\,\mathcal{R}(M)^{-ab}_{cd}\,\mathcal{R}(M)^{-ab}_{cd} \\
&\quad +32\, R(\mathcal{V})^{-k}_{ab}{}_l\, R(\mathcal{V})^{-l}_{ab}{}_k \\
&\quad -32\, T^{ij\,ab}\, D_a\, D^c T_{cb\,ij} \\
&\quad +128\,\bar{\mathcal{R}}(S)^{ab}_i\, R(Q)^i_{ab} \\
&\quad +384\,\bar{R}(Q)^{ab\,i}\gamma_a D_b\chi_i\,.
\end{aligned} \tag{2.4}$$

We refer to [1] for a precise definition of the various curvature tensors. The derivatives D_a are superconformally covariant.

In the presence of a chiral background superfield, the coupling of the abelian vector multiplets to the Weyl multiplet is encoded in a function $F(X^I, \hat{A})$, which is holomorphic and homogenous of degree two,

$$\begin{aligned}
& X^I F_I + w\hat{A}\, F_{\hat{A}} = 2F\,, \\
& F_I = \partial_{X^I} F\,, \quad F_{\hat{A}} = \partial_{\hat{A}} F\,.
\end{aligned} \tag{2.5}$$

The field equations of the vector multiplets are subject to equivalence transformations corresponding to electric-magnetic duality, which will not involve the fields of the Weyl multiplet and of the chiral background. As is well-known, two complex $(2n+2)$-component vectors can be defined which transform linearly under the $\mathrm{SP}(2n+2;\mathbf{R})$ duality group, namely

$$\begin{pmatrix} X^I \\ F_I(X,\hat{A}) \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} F^{+I}_{\mu\nu} \\ G^+_{\mu\nu\,I} \end{pmatrix}\,. \tag{2.6}$$

The first vector has weights $w=1$ and $c=-1$, whereas the second one has zero Weyl and chiral weights. The field strengths $G^{\pm}_{\mu\nu\,I}$ are defined as follows:

$$\begin{aligned}
G^+_{\mu\nu I} &= \bar{F}_{IJ}\, F^{+J}_{\mu\nu} + \mathcal{O}^+_{\mu\nu I}\,, \\
G^-_{\mu\nu I} &= F_{IJ} F^{-J}_{\mu\nu} + \mathcal{O}^-_{\mu\nu I}\,,
\end{aligned} \tag{2.7}$$

where

$$\mathcal{O}^+_{\mu\nu I} = \tfrac{1}{4}(F_I - \bar{F}_{IJ} X^J)\, T_{\mu\nu ij}\varepsilon^{ij} + \hat{F}^+_{\mu\nu}\,\bar{F}_{I\hat{A}}\,. \tag{2.8}$$

They appear in the field equations of the vector fields. Eventually we will solve the Bianchi identities, $\mathcal{D}^\mu(F^- - F^+)^I_{\mu\nu} = 0$, and the field equations, $\mathcal{D}^\mu(G^- - G^+)^I_{\mu\nu} = 0$, for a given configuration of magnetic and electric charges in a static spacetime geometry with the chiral background turned on. These charges, which will be denoted by (p^I, q_I), comprise a symplectic vector.

Next, let us introduce a particular spinor that transforms inhomogenously under S-supersymmetry transformations. This spinor is given by

$$\begin{aligned}
\zeta^V_i &\equiv -\Big(\Omega^I_i\,\frac{\partial}{\partial X^I} + \hat{\Psi}_i\,\frac{\partial}{\partial\hat{A}}\Big)\mathcal{K} \qquad (2.9) \\
&= -i\,\mathrm{e}^{\mathcal{K}}\Big[(\bar{F}_I - \bar{X}^J F_{IJ})\Omega^I_i - \bar{X}^I F_{I\hat{A}}\,\hat{\Psi}_i\Big]\,,
\end{aligned}$$

where we introduced the symplectically covariant factor (with $w=2$ and $c=0$),

$$\mathrm{e}^{-\mathcal{K}} = i\Big[\bar{X}^I\, F_I(X,\hat{A}) - \bar{F}_I(\bar{X},\bar{\hat{A}})\, X^I\Big]\,, \tag{2.10}$$

which resembles (but is not equal to) the Kähler potential in special geometry. It can be shown, using the results contained in [5], that ζ^V_i transforms covariantly under symplectic reparametrizations. Under Q- and S-supersymmetry ζ^V_i transforms as (ignoring higher-order fermionic terms)

$$\begin{aligned}
\delta\zeta^V_i &= \mathrm{e}^{\mathcal{K}}\not{\mathcal{D}}\mathrm{e}^{-\mathcal{K}}\epsilon_i + 2i\not{\mathcal{A}}\epsilon_i - \tfrac{1}{2} i\varepsilon_{ij}\,\mathcal{F}^-_{\mu\nu}\,\gamma^{\mu\nu}\epsilon^j \\
&\quad +\mathrm{e}^{\mathcal{K}}\Big[(\bar{F}_I - \bar{X}^J\bar{F}_{IJ}) N^{IK} F_{KA}\,\hat{B}_{ij} \\
&\quad -(\bar{F}_I - \bar{X}^J F_{IJ}) N^{IK}\bar{F}_{KA}\,\varepsilon_{ik}\varepsilon_{jl}\hat{B}^{kl}\Big]\,\epsilon^j \\
&\quad +2\,\eta_i\,,
\end{aligned} \tag{2.11}$$

where

$$\begin{aligned}
\mathcal{A}_\mu &= \tfrac{1}{2}\,e^{\mathcal{K}}\,\Big(\bar{X}^J\,\overleftrightarrow{\mathcal{D}}_\mu\, F_J - \bar{F}_J\,\overleftrightarrow{\mathcal{D}}_\mu\, X^J\Big)\,, \\
\mathcal{F}^-_{\mu\nu} &= \mathrm{e}^{\mathcal{K}}\,\Big(\bar{F}_I\, F^{-I}_{\mu\nu} - \bar{X}^I\, G^-_{\mu\nu\,I}\Big)\,.
\end{aligned} \tag{2.12}$$

In arriving at (2.11) we have used the field equations for the auxiliary fields Y^I_{ij} [5], which is necessary for $\delta\zeta^V_i$ to take a symplectically covariant form.

We note that $\delta\zeta_i^V$ is not the only spinor that can be constructed which transform inhomogenously under S-supersymmetry transformations. Another such spinor, which we denote by ζ_i^H, is constructed out of the hypermultiplet fermions [1]. It transforms as follows under Q- and S-supersymmetry,

$$\delta\zeta_i^{\rm H} = \tfrac{1}{2}\chi^{-1}\not{D}\chi\,\varepsilon_{ij}\,\epsilon^j + \not{k}_{ij}\,\epsilon^j + \varepsilon_{ij}\,\eta^j \,, \quad (2.13)$$

where χ denotes the hyper-Kähler potential and where $k_{\mu ij}$ denotes a quantity that is symmetric in i,j [1] , but whose explicit form is not important here.

Since the ζ_i transform inhomogenously under S-supersymmetry, they can act as compensators for this symmetry. This observation is relevant when constructing supersymmetric backgrounds, where one requires (some of) the Q-supersymmetry variations of the spinors (as well as of derivatives of the spinors) to vanish modulo a uniform S-transformation. This can conveniently be done by considering S-invariant spinors, constructed by employing ζ_i. Relevant examples of such spinors are, for instance, $\Omega_i^I - X^I\zeta_i^V$ and $\hat\Psi_i - w\hat A\,\zeta_i^V$.

3. The ansatz

We seek to construct static multi-center black hole solutions with $N = 1$ residual supersymmetry. For the line element we make the ansatz

$$ds^2 = -e^{2g(\vec x)}dt^2 + e^{2f(\vec x)}d\vec x^2 \quad . \qquad (3.1)$$

We impose the following restriction on the Q-supersymmetry transformation parameter ϵ_i,

$$\epsilon_i = h\,\varepsilon_{ij}\,\gamma_0\,\epsilon^j \quad , \qquad (3.2)$$

where $h(\vec x)$ denotes a phase factor of chiral weight $c = 1$. The condition (3.2) is covariant with respect to SU(2) and spatial rotations. The multi-center solutions that we wish to construct have the feature that, when the centers are made to coincide, they lead to one-center solutions that are invariant with respect to SU(2) and spatial rotations. The latter satisfy condition (3.2).

In addition to (3.2), we impose that $\mathcal{A}_\mu = 0$ as well as $\mathcal{F}^-_{\mu\nu} = 0$.

We denote the magnetic and electric charges associated to each center by (p^I_A, q_{AI}). In the geometry (3.1) the Bianchi identities and field equations for the vector fields are solved by

$$\begin{aligned} F^{-I}_{ti} - F^{+I}_{ti} &= -i\,e^{g-f}\partial_i H^I \ , \\ G^-_{tiI} - G^+_{tiI} &= -i\,e^{g-f}\partial_i H_I \ , \end{aligned} \qquad (3.3)$$

where H^I and H_I denote harmonic functions given by

$$\begin{aligned} H^I &= \sum_A \left(h^I_A + \frac{p^I_A}{|\vec x - \vec x_A|}\right), \\ H_I &= \sum_A \left(h_{AI} + \frac{q_{AI}}{|\vec x - \vec x_A|}\right) \ . \end{aligned} \qquad (3.4)$$

Here h^I_A and h_{AI} denote integration constants. We now identify $\hat A$ with $(\varepsilon_{ij}T^{abij})^2$ so that we are dealing with black hole solutions in the presence of R^2-interactions.

4. Static multi-center solutions

It will be convenient [3] to use rescaled variables $Y^I = \mathrm{e}^{\mathcal{K}/2}\bar\Sigma X^I$ and $\Upsilon = \mathrm{e}^{\mathcal{K}}\bar\Sigma^2\hat A$. Here $\bar\Sigma$ is taken to have weights $c = 1$ and $w = 0$ so that Y^I and Υ have vanishing chiral and Weyl weights. Then, from (2.10) and from (2.12), we obtain

$$\begin{aligned} |\Sigma|^2 &= i\Big[\bar Y^I\,F_I(Y,\Upsilon) - \bar F_I(\bar Y,\bar\Upsilon)\,Y^I\Big] \ , \\ A_\mu &= \tfrac{i}{2}\partial_\mu \log\tfrac{\Sigma}{\bar\Sigma} - \mathcal{A}^Y_\mu \ , \\ \mathcal{A}^Y_\mu &= \tfrac{1}{2}\,\tfrac{1}{|\Sigma|^2}\left(\bar Y^J\,\overset{\leftrightarrow}{\partial}_\mu\,F_J - \bar F_J\,\overset{\leftrightarrow}{\partial}_\mu\,Y^J\right) \ . \end{aligned} \qquad (4.1)$$

Using (3.1), we find [1] that the vanishing of the Q-supersymmetry variation (subject to (3.2)) of the various S-invariant spinors yields a number of restrictions on the $N = 1$ background, as follows. In a Poincaré frame (where $b_\mu = 0$ and $\mathcal{K} =$ const.) we find that

$$\begin{aligned} e^{-2g} &= e^{2f} = \mathrm{e}^{\mathcal{K}}|\Sigma|^2 \ , \\ \mathrm{e}^{\mathcal{K}/2}\bar\Sigma T^-_{ti} &= 4\,\partial_i f \ , \quad \Upsilon = -64(\partial_i f)^2 \ , \\ F^I_{ti} &= -\partial_i\Big[e^{-2f}(Y^I + \bar Y^I)\Big] \ , \\ \mathcal{A}^Y_\mu &= 0 \ . \end{aligned} \qquad (4.2)$$

The symplectic vector $(Y^I, F_I(Y,\Upsilon))$ is determined in terms of the symplectic vector (H^I, H_I)

as follows:

$$\partial_i \begin{pmatrix} Y^I - \bar{Y}^I \\ F_I(Y,\Upsilon) - \bar{F}_I(\bar{Y},\bar{\Upsilon}) \end{pmatrix} = i\,\partial_i \begin{pmatrix} H^I \\ H_I \end{pmatrix} . \tag{4.3}$$

Thus, we see that as one approaches the individual centers ($|\vec{x}-\vec{x}_A| \to 0$) the scalar fields Y^I are entirely determined in terms of the charges associated to these individual centers. This behaviour, namely that the values of the Y^I near the centers are independent of the constants h^I_A and h_{AI} which determine the values of the Y^I far away from the centers, is the same that has been observed without R^2-interactions [6].

In addition, we find that $h = \bar{\Sigma}/|\Sigma|$, whereas the field D is determined to be $D = -\frac{1}{3}R$. The SU(2) curvature $R(\mathcal{V})_{\mu\nu}{}^i{}_j$, and hence $\hat{B}_{ij}$, vanishes. Since $\mathcal{A}^Y_\mu = 0$, also the U(1) curvature $R(A)_{\mu\nu}$ vanishes.

The solution given above describes a static multi-center black hole with residual $N=1$ supersymmetry in the presence of R^2-interactions. It approaches flat Minkowski spacetime at spatial infinity. Setting $\mathrm{e}^{\mathcal{K}}|\Sigma|^2_\infty = 1$ expresses its ADM mass as $M_{\rm ADM} = \mathrm{e}^{\mathcal{K}} \sum_A (p^I_A F_I(Y_\infty) - q_{AI} Y^I_\infty)$.

In the case of one center, the solution interpolates between two $N=2$ supersymmetric vacua [7]: flat spacetime at spatial infinity and Bertotti-Robinson spacetime at the horizon. When switching off R^2-interactions this solution agrees with the one constructed in [8]. Its macroscopic entropy is given by [3]

$$\mathcal{S} = \pi r^2 \Big[|\Sigma|^2 + 4\,\mathrm{Im}\Big(\Upsilon\, F_\Upsilon(Y,\Upsilon)\Big)\Big]\Big|_{r=0} . \tag{4.4}$$

5. Outlook

The static multi-center solution (4.2) can now be used as the starting point for computing the metric on the moduli space of four-dimensional BPS black holes in the presence of R^2-interactions. In the absence of R^2-interactions, it was found [9, 10] that the moduli metric of electrically charged BPS black holes is determined in terms of a moduli potential μ given by $\mu = \int d^3x\, e^{4f}$. It was furthermore established [9, 10] that for small black hole separations the associated one-dimensional Lagrangian describing the slow-motion of these BPS black holes exhibits an enhanced superconformal symmetry. It was suggested [9] that it should be possible to reproduce the macroscopic entropy of BPS black holes by performing a state counting in this superconformal quantum mechanics model. This would imply that the degeneracy of states of such a model is encoded in the moduli potential μ. In view of the formula (4.4) for the macroscopic entropy one thus expects that μ will receive corrections steming from R^2-interactions. This is indeed likely to be the case, since the one-dimensional Lagrangian describing the slow-motion of the black holes will now be derived from a four-dimensional action containing R^2-interactions. Schematically, since the four-dimensional Einstein-Maxwell action gives rise to a moduli potential $\mu = \int d^3x\, e^{4f}$, the term $\mathrm{Im} F_{\hat{A}}\, R\, |T|^2$, which appears in the four-dimensional Lagrangian with R^2-interactions, suggests a correction to the moduli potential of the form $\int d^3x\, e^{2f}\,\mathrm{Im}(\Upsilon F_\Upsilon)$. In view of (4.4), this suggests that in the presence of R^2-terms the moduli potential μ will be given by $\mu = \int d^3x\, e^{2f}[e^{2f} + 4\,\mathrm{Im}(\Upsilon\, F_\Upsilon(Y,\Upsilon))]$. This feature is currently under investigation [11].

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